

International School **L**Andslide **R**isk **A**ssessment and **M**itigation  
LARAM School 2007 (7-22 September, Ravello, Italy)  
Session 1: Introduction to landslides: Landslide analysis using approaches based on:  
Geology, Geotechnics and Geomechanics

## **Basic Geodynamics of Landslides: II. The Dynamics of Landslide Run-out**

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(<http://geolab.mechan.ntua.gr>)



**The dynamic slip circle method**

**The dynamics of landslide run-out (lagrangean)**

**Flow-slides**

## II. The Dynamics of Landslide Run-out

Davis, H.T., *Introduction to Nonlinear Differential and Integral Equations*, Dover, 1962.

Hauger., W, Schnell, W. und Gross, D., *Technische Mechanik, Bd. 3:Kinetik*, Springer Verlag, 1989.

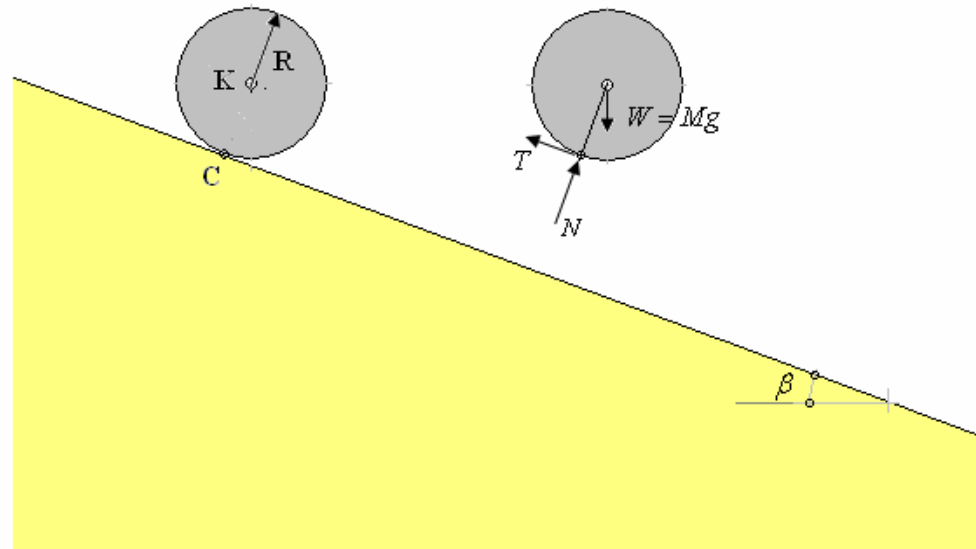
Heim, A.. *Bergsturtz und Menschenleben*. Fretz u. Wasmuth, Zürich, 1932.

## Rolling and sliding

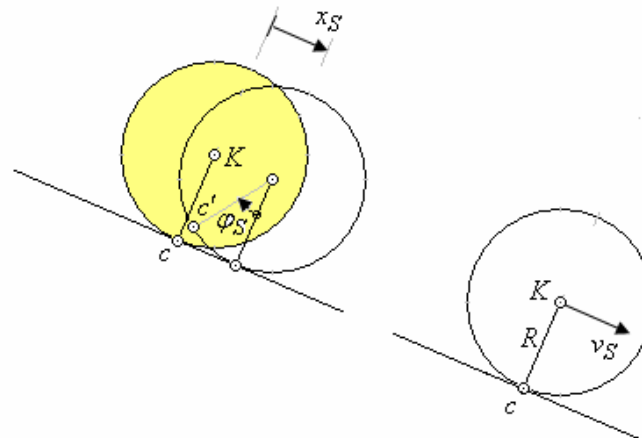
$$M\ddot{x}_S = W \sin \beta - T$$

$$0 = -W \cos \beta + N$$

$$\Theta_S \ddot{\varphi}_S = RT$$



Rolling:  $x_S = R\varphi_S$

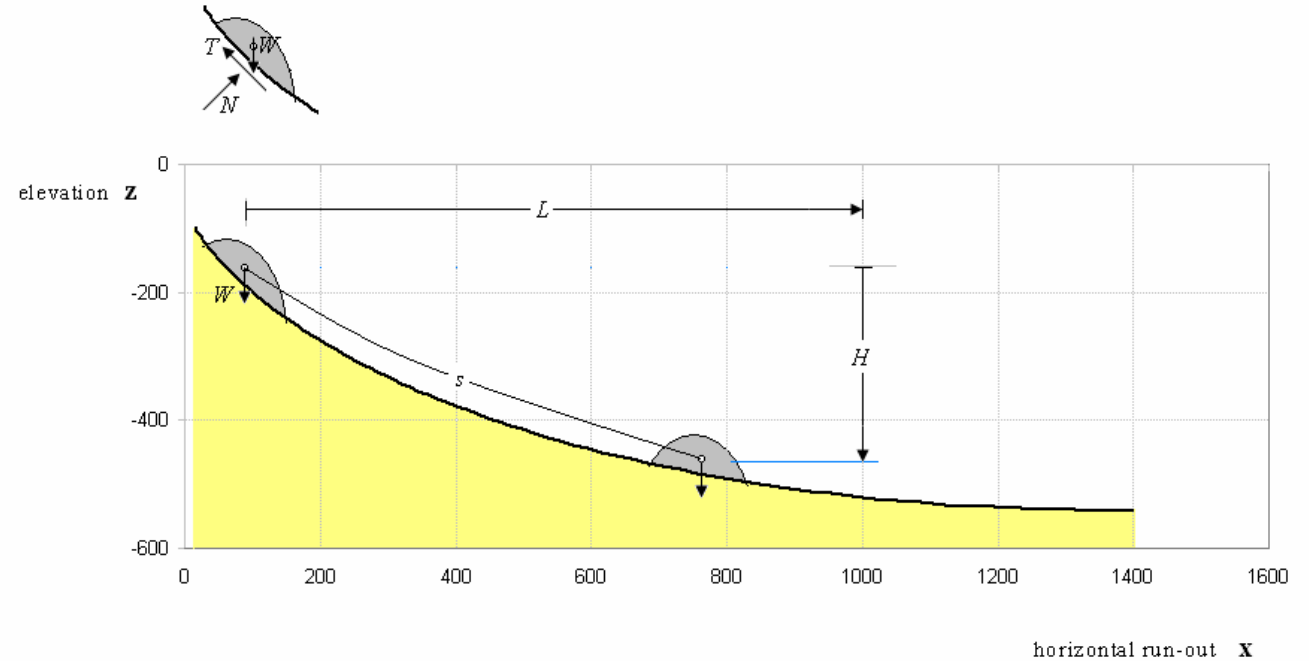


$3.5\mu < \tan \beta \Rightarrow \text{rolling} + \text{sliding}$

$\mu < \tan \beta \leq 3.5\mu \Rightarrow \text{rolling}$



# Run-out distance



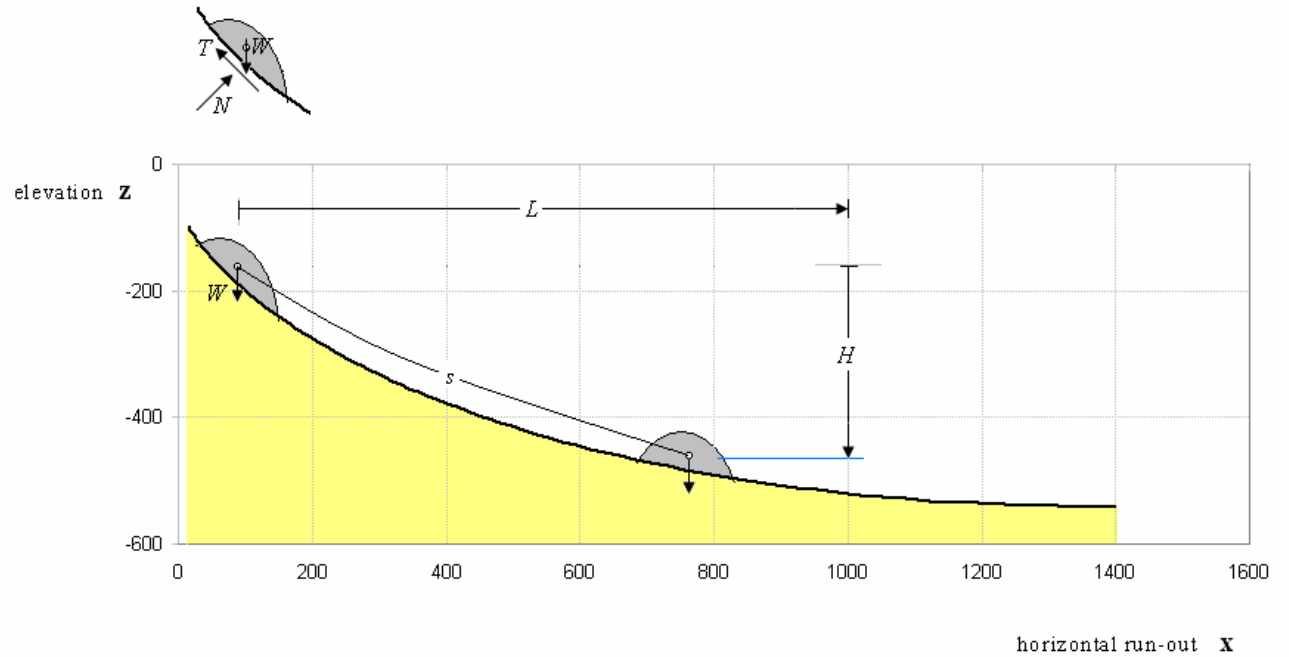
$$d\left(\frac{1}{2}MV^2\right) = Wdz - dD \Rightarrow$$

$$M \int_0^S VdV = W \int_0^H dz - \int_0^S dD \Rightarrow WH = \int_0^S Tds$$

$$WH = \int_0^S T ds$$

$$T = N\mu$$

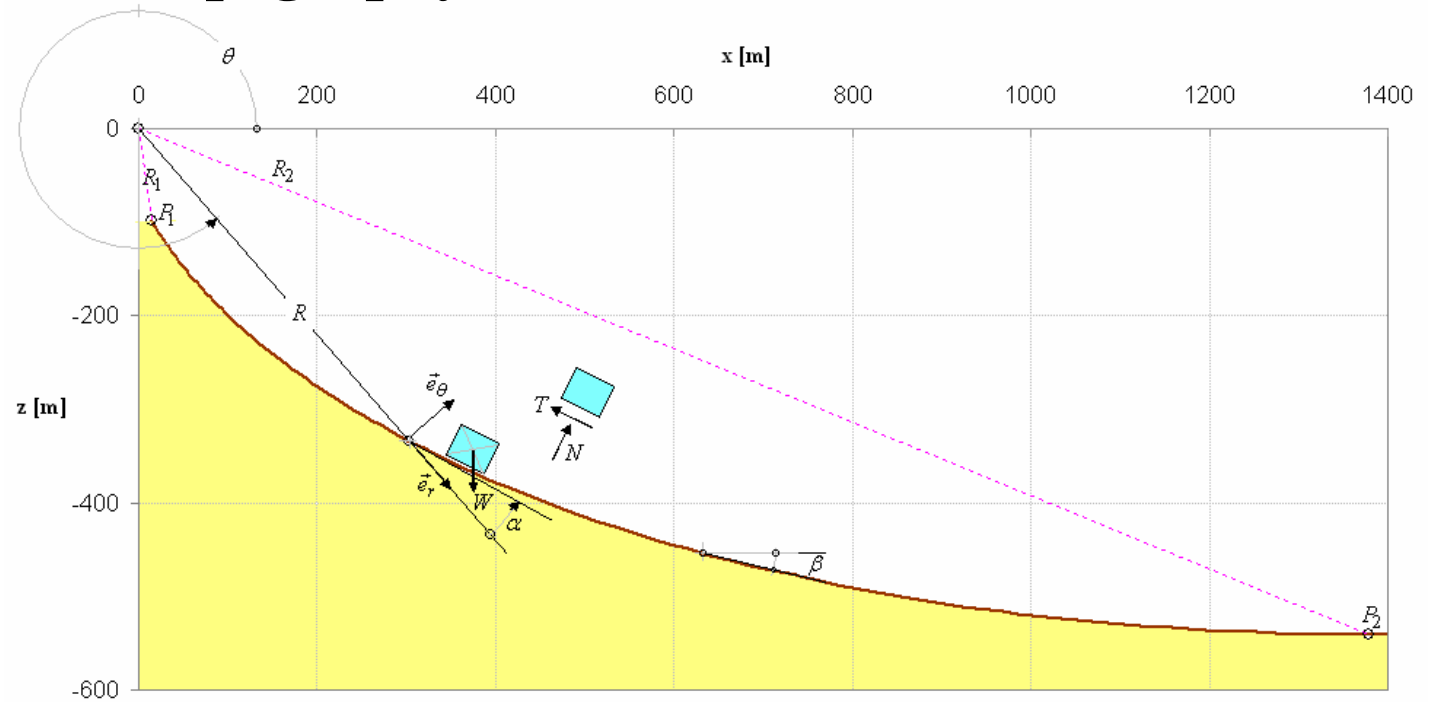
$$N \approx W \cos \beta$$



$$WH = \mu W \int_0^S \cos \beta ds = \mu W \int_0^L dx = \mu WL \Rightarrow$$

$$\frac{H}{L} = \mu$$

# Sliding on variable topography



$$W_r + N_r + T_r = Ma_r \quad , \quad a_r = \ddot{R} - R\dot{\theta}^2$$

$$W_\theta + N_\theta + T_\theta = Ma_\theta \quad , \quad a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta}$$



## Log-spiral track

$$k\ddot{\theta} + (k^2 - 1)\dot{\theta}^2 = -\omega^2 e^{-k\theta} (\sin \theta + n \sin(\alpha + \varphi))$$

$$\ddot{\theta} + 2k\dot{\theta}^2 = -\omega^2 e^{-k\theta} (\cos \theta - n \cos(\alpha + \varphi))$$

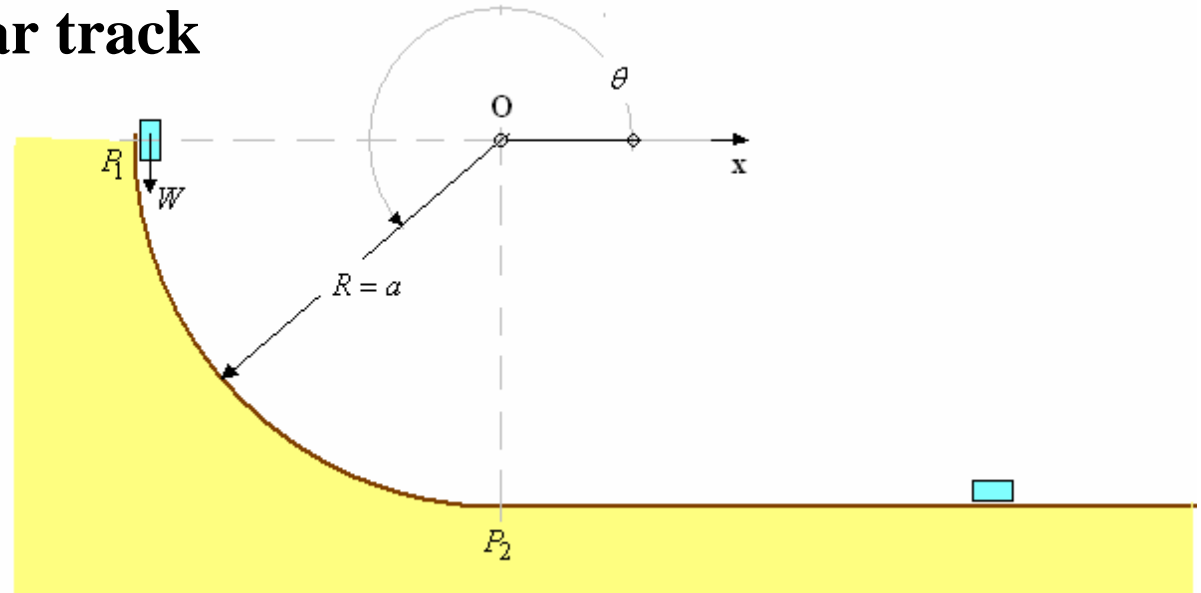
$$\omega^2 = \frac{g}{a}, \quad n = \frac{1}{\cos \varphi} \frac{N}{W}$$

$$k = \cot \alpha$$

$\alpha$  angle between the radius and the tangent to the log-spiral

$\alpha = \pi / 2 \Rightarrow$  circle

# The frictionless circular track



$$k = 0, \quad \varphi = 0 \quad \Rightarrow \quad \begin{cases} \dot{\theta}^2 = \omega^2 (\sin \theta + n) \\ \ddot{\theta} = -\omega^2 \cos \theta \end{cases}$$

$$n = \frac{N}{W}$$



## Frictionless track

$$\begin{cases} \dot{\theta}^2 = \omega^2 (\sin \theta + n) \\ \ddot{\theta} = -\omega^2 \cos \theta \end{cases}$$

$$\ddot{\theta} = \frac{d\dot{\theta}}{d\theta} \frac{d\theta}{dt} = \dot{\theta} \frac{d\dot{\theta}}{d\theta} = -\omega^2 \cos \theta \Rightarrow \dot{\theta} d\dot{\theta} = -\omega^2 \cos \theta d\theta$$

$$\Rightarrow \frac{1}{2} \dot{\theta}^2 = -\omega^2 \sin \theta$$

$$\Rightarrow \omega^2 (\sin \theta + n) = 2\omega^2 (-\sin \theta) \Rightarrow n = 3(-\sin \theta)$$

the slope angle at any point of the circular track is:

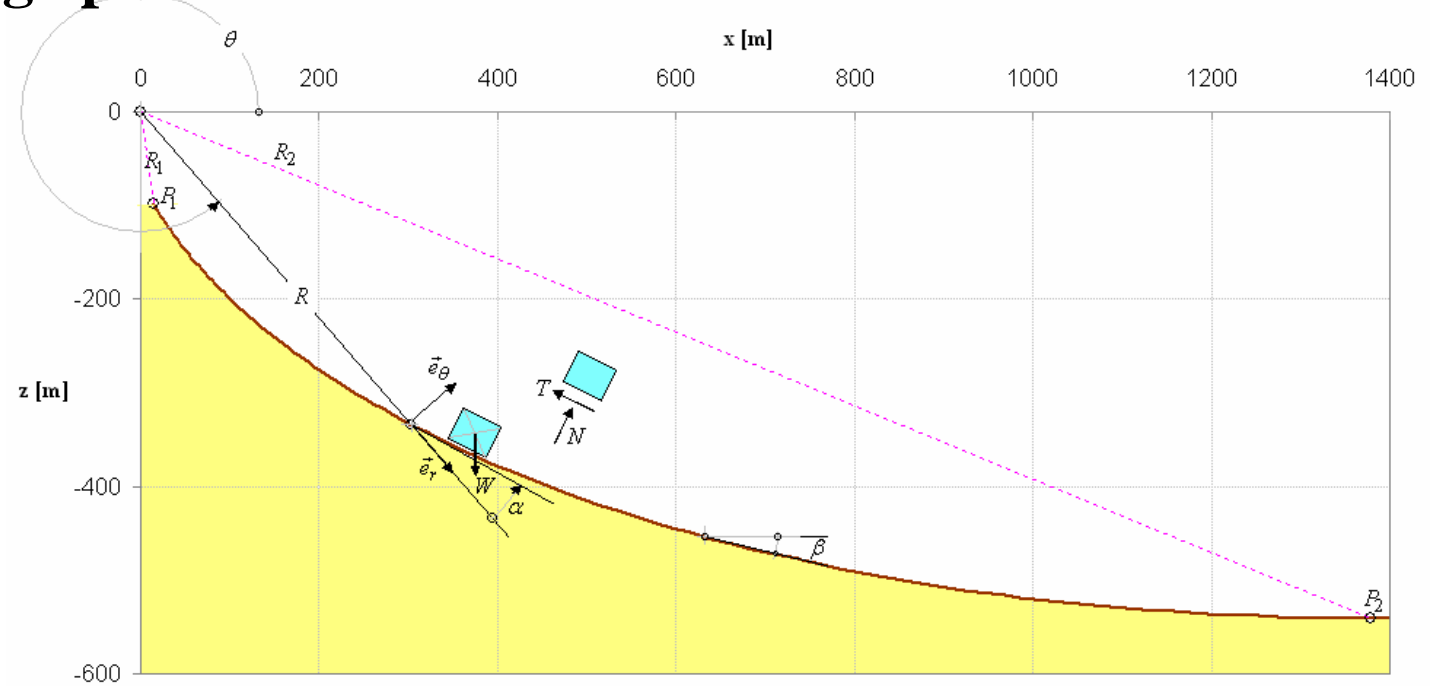
$$\beta = 2\pi - (\pi/2 + \theta) = 3\pi/2 - \theta \Rightarrow n = 3 \cos \beta$$

The normal reaction amplification is due to the centripetal acceleration and assumes its maximum value at the lowest point of the track,

$$\beta = 0 \quad \Rightarrow \quad n = n_{\max} = 3$$

This value is **three** times higher than the static value! Thus the transition from the curved track to a planar one is accompanied always with a sudden drop on the normal reaction force.

# The frictional log-spiral track



$$A_0 \ddot{\theta} + A_1 \dot{\theta}^2 + A_2 e^{-k\theta} \sin(\theta + \alpha + \varphi) = 0$$

$$A_1 = k + \tan(\alpha + \varphi)$$

$$A_1 = k^2 + 2k \tan(\alpha + \varphi) - 1$$

$$A_2 = \omega^2 / \cos(\alpha + \varphi)$$

$$n = -\frac{1}{\sin(\alpha + \varphi)} \left( \omega^{-2} e^{k\theta} \left( k\ddot{\theta} + (k^2 - 1)\dot{\theta}^2 \right) + \sin \theta \right)$$

## Numerical integration of the governing non-linear o.d. equation (curvature method)

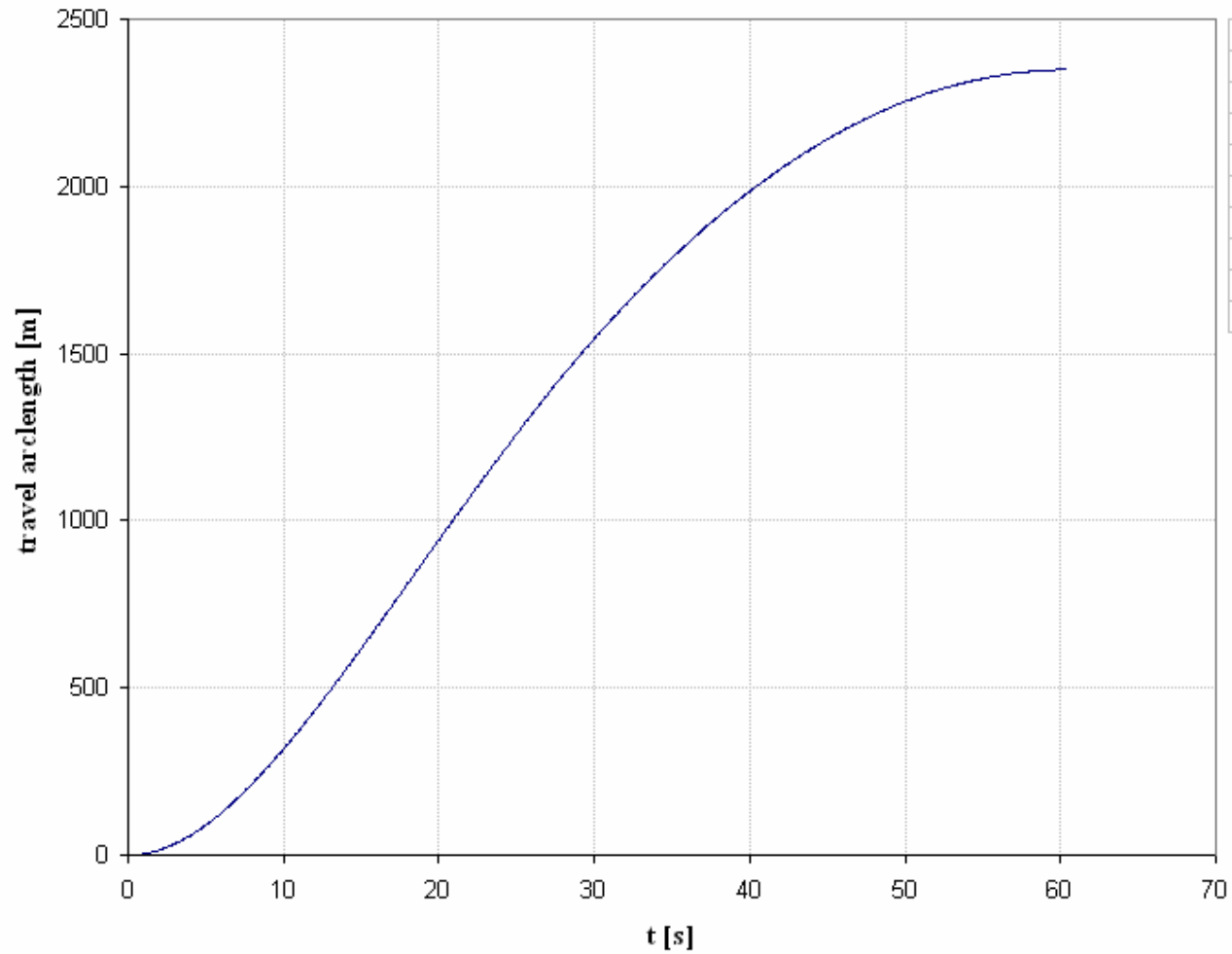
$$\ddot{\theta} = f(t, \theta, \dot{\theta}) \quad (1)$$

$$\theta(0) = \theta_0 \quad , \quad \dot{\theta}(0) = \dot{\theta}_0 \quad (2)$$

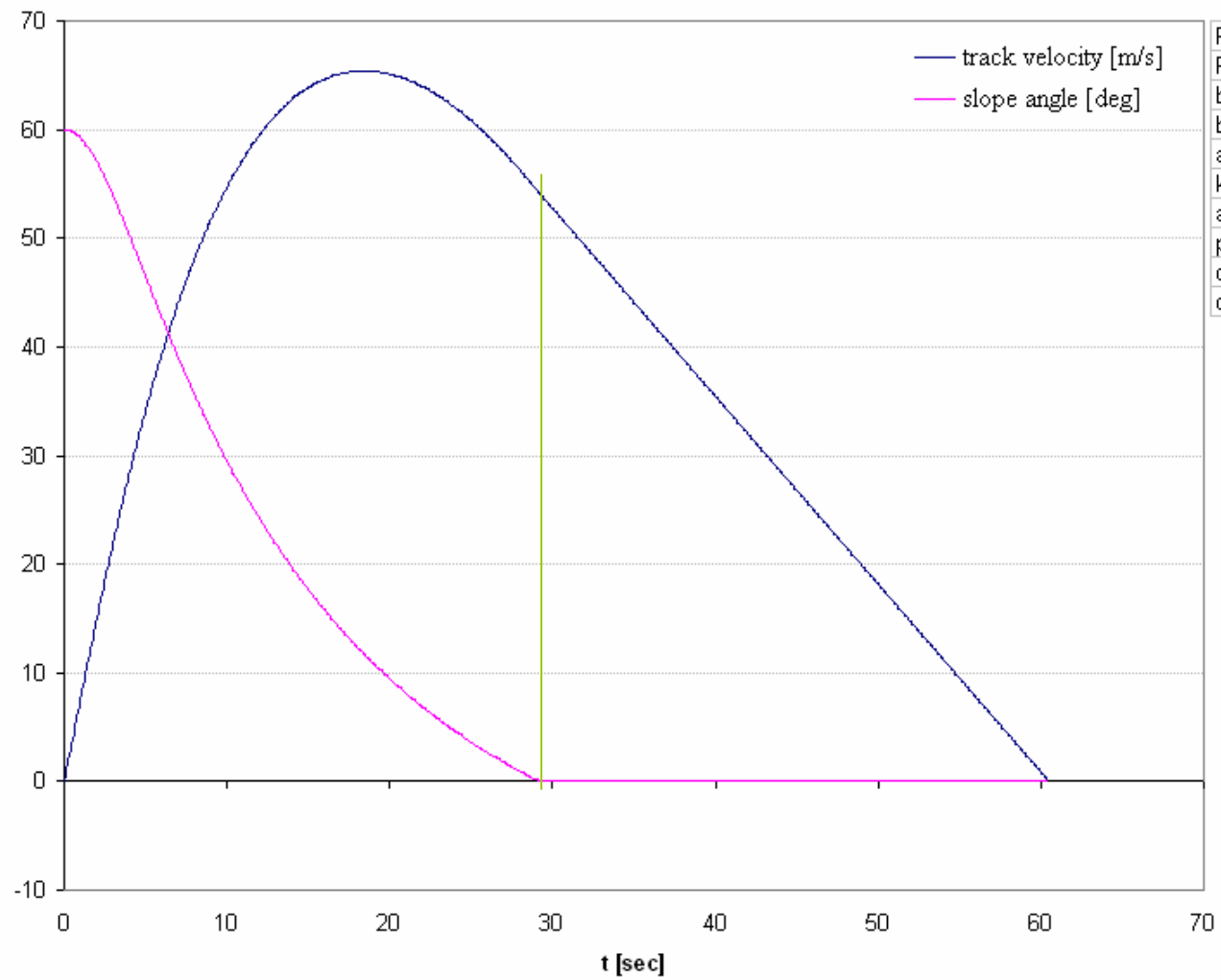
$$(1) \Rightarrow \ddot{\theta}(0) = f(t_0, \theta_0, \dot{\theta}_0)$$

$$\theta_1 = \theta_0 + \dot{\theta}_0 \Delta t + \frac{1}{2} \ddot{\theta}_0 \Delta t^2 \quad ; \quad \dot{\theta}_1 = - \frac{\ddot{\theta}_0 \Delta t + \dot{\theta}_0 (1 + \dot{\theta}_0^2)}{\ddot{\theta}_0 \left( \dot{\theta}_0 \Delta t + \frac{1}{2} \ddot{\theta}_0 \Delta t^2 \right) - (1 + \dot{\theta}_0^2)}$$

$$(1) \Rightarrow \ddot{\theta}_1 = f(t_1, \theta_1, \dot{\theta}_1)$$

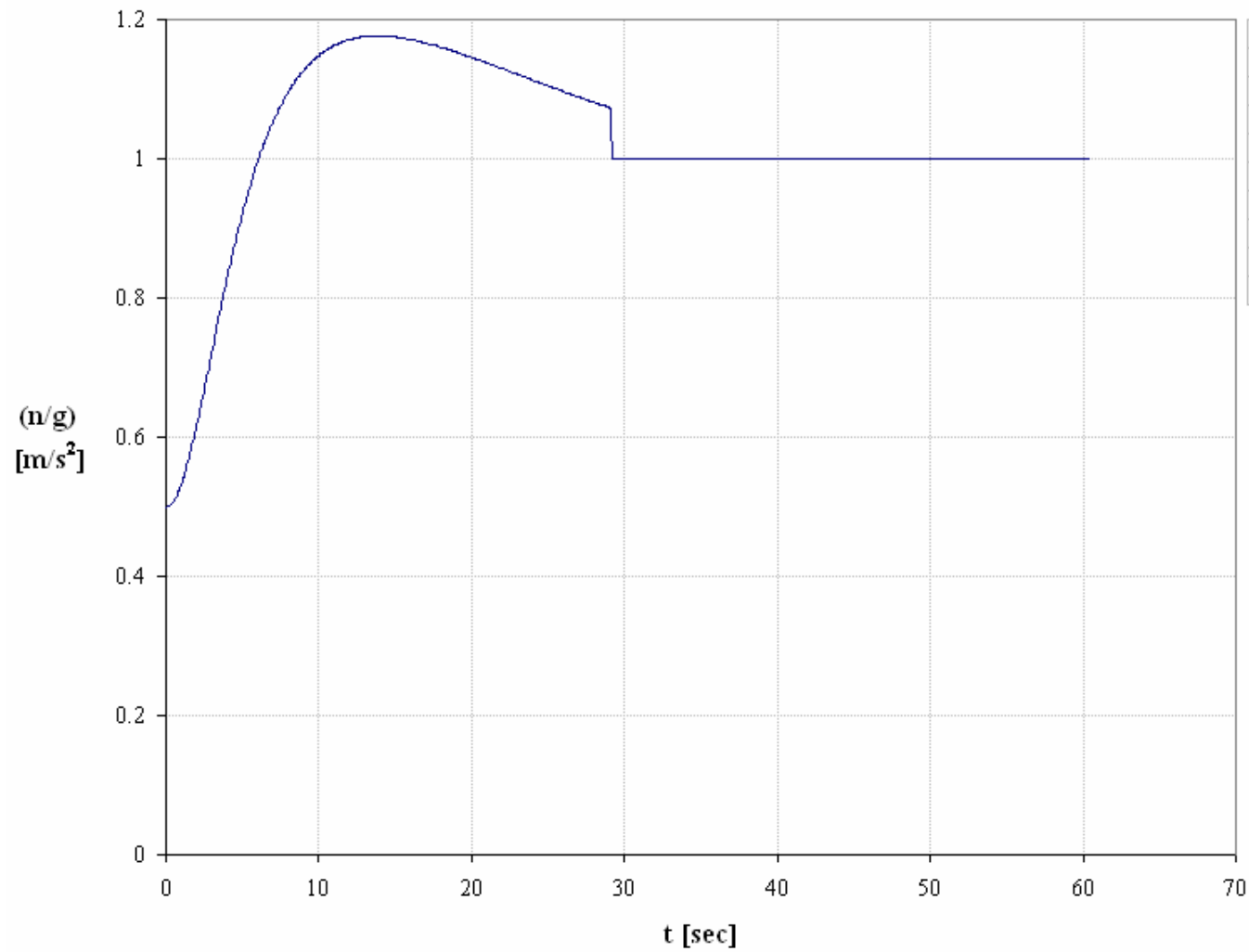


R1=	100	[m]
R2=	1500	[m]
beta1=	60	[o]
beta2=	0	[o]
alpha=	21.14	[o]
k=	2.59E+00	[-]
a=	3.42E-04	[m]
phi=	10	[o]
omega=	1.69E+02	[1/sec^2]
dt=	1.00E-06	[sec]

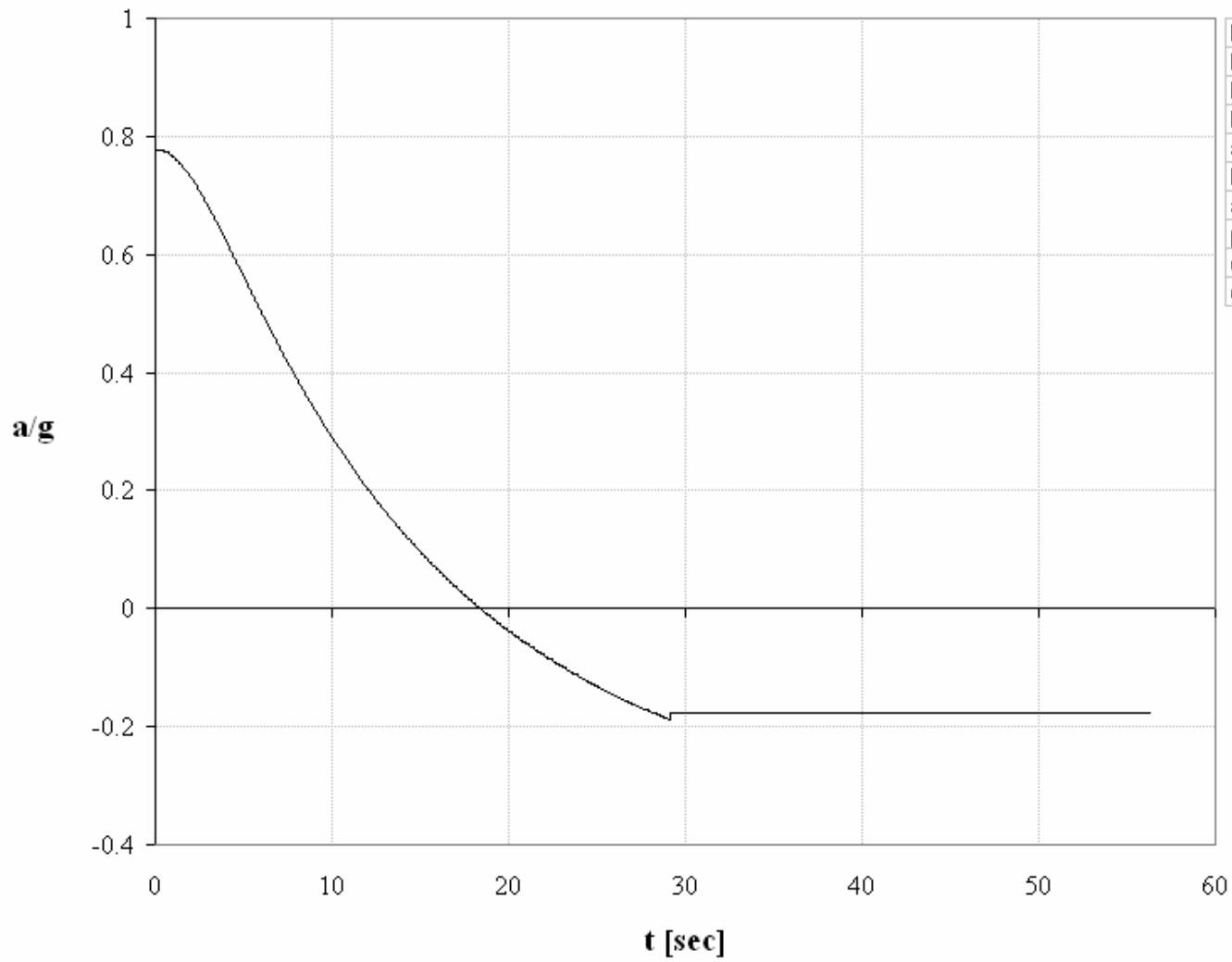


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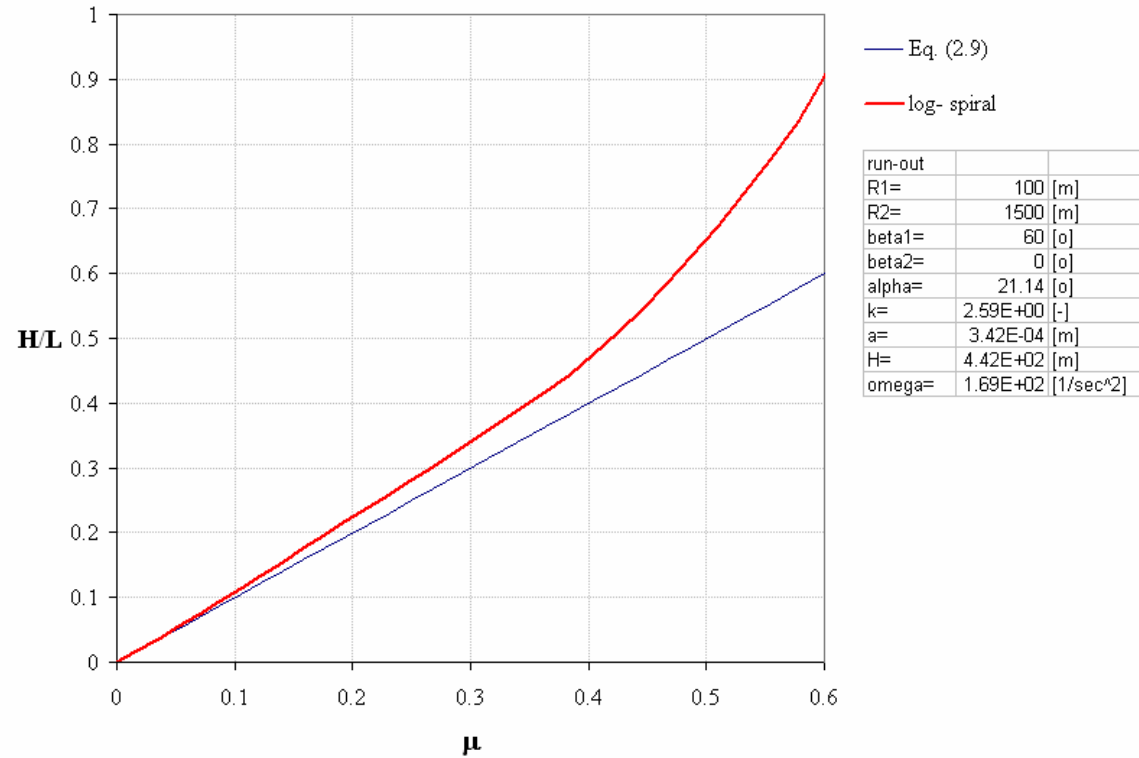
R1=	100	[m]
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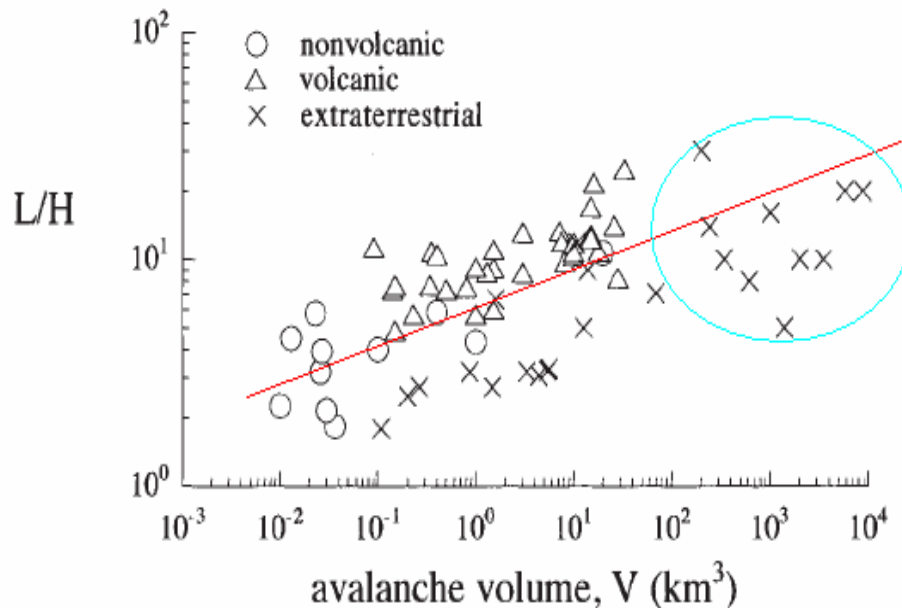
# Discussion

$$L \approx \frac{H}{\mu}$$



**is a rather good estimate for low basal friction for the sliding block run-out distance**

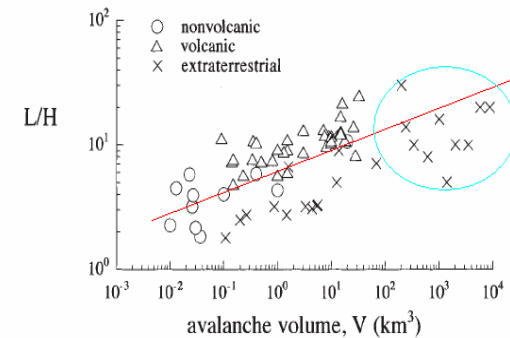
As was first noticed by Heim (1932), catastrophic landslides are characterized by a very low  $\mu$ -value, thus indicating a very small true friction coefficient. Indeed field data suggest a reduction of the friction coefficient,  $\mu$ , with the volume of the landslide.



$$\frac{L}{H} = \frac{1}{\mu}$$

Figure 1. Relative runout  $L/H$  as function of rockfall volume  $V$ . Data compiled from Howard (1973), Voight (1978), Lucchitta (1978, 1979), Crandell et al. (1984), Francis et al. (1985), Siebert et al. (1987), McEwen (1989), and Stoope and Sheridan (1992).

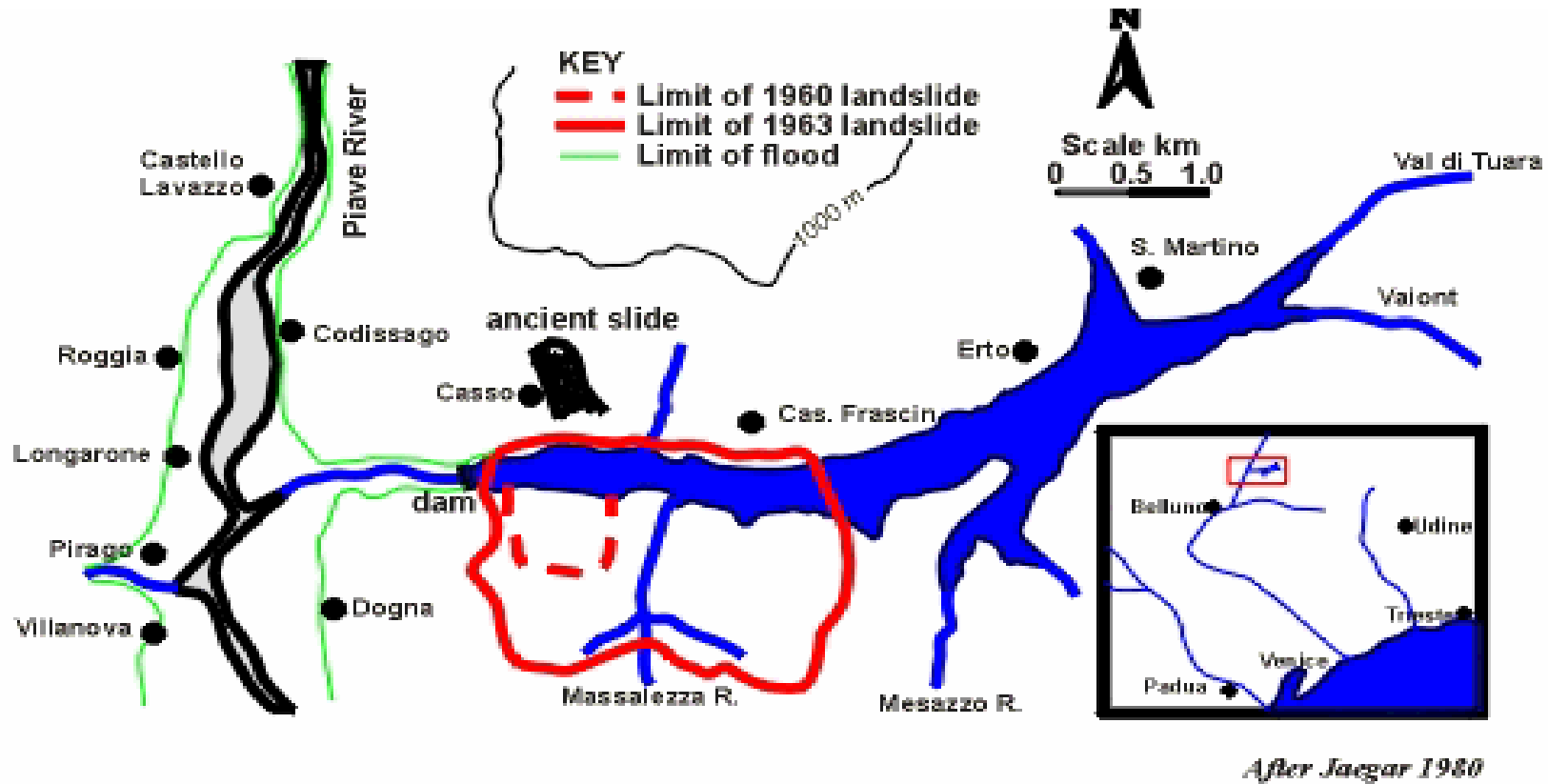
Field data suggest a reduction of the friction coefficient,  $\mu$ , with the volume of the landslide. This observation has triggered intensive research efforts for the disclosure of possible mechanisms that would explain this severe reduction in frictional resistance. Recent studies on catastrophic landslides corroborate the original "vaporization" concept of Habib (1967, 1975). The idea that a heat generating mechanism might account for the total loss of strength of large earth slides due to thermal pressurization of the pore fluid inside the failure zone has been discussed in the past by Uriel & Molina (1974), Goguel (1978), Anderson (1980), Voight & Faust (1982), Vardoulakis (2000, 2002) and Veveakis & Vardoulakis (2007), Goren & Aharonov (2007), and Blasio (2008).



$$\frac{L}{H} = \frac{1}{\mu}$$

Figure 1. Relative runout  $L/H$  as function of rockfall volume  $V$ . Data compiled from Howard (1973), Voight (1978), Lucchitta (1978, 1979), Crandell et al. (1984), Francis et al. (1985), Siebert et al. (1987), McEwen (1989), and Stoores and Sheridan (1992).

# The Vajont landslide



(<http://www.land-man.net/vajont/vajont.html>)

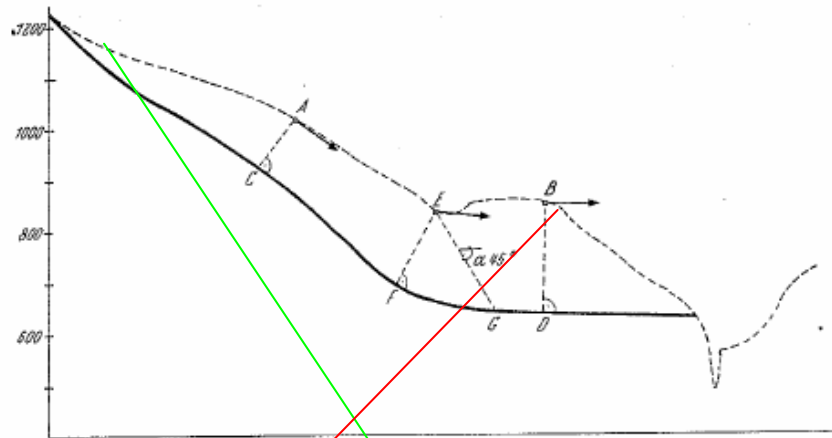


Fig. 9. Inclination of the displacement vectors  
 Explanation: see text

## The Vaiont slide (2006)



**Vaiont** is located in the south-eastern part of the Dolomite Region of the Italian Alps, about 100 km north of Venice. At 22:38 GMT on October 9 1963 catastrophic failure of the landslide occurred. **The entire mass slid approximately 500 m northwards at up to 30 m/sec.** The mass completely blocked the gorge to a depth of up to 400m , and it traveled up to 140 m up the opposite bank. **Movement of the landslide mass ceased after a maximum of 45 sec.** At the time the reservoir contained 115 million m<sup>3</sup> of water. A wave of water was pushed up the opposite bank and destroyed the village of Casso, 260 m above lake level before over-topping the dam by up to 245 m. The water, estimated to have had a volume of about 30 million m<sup>3</sup>, then fell more than 500 m onto the villages of Longarone, Pirago, Villanova, Rivalta and Fae, totally decimating them. **A total 2500 lives were lost.** However the dam was not destroyed and is still standing today.

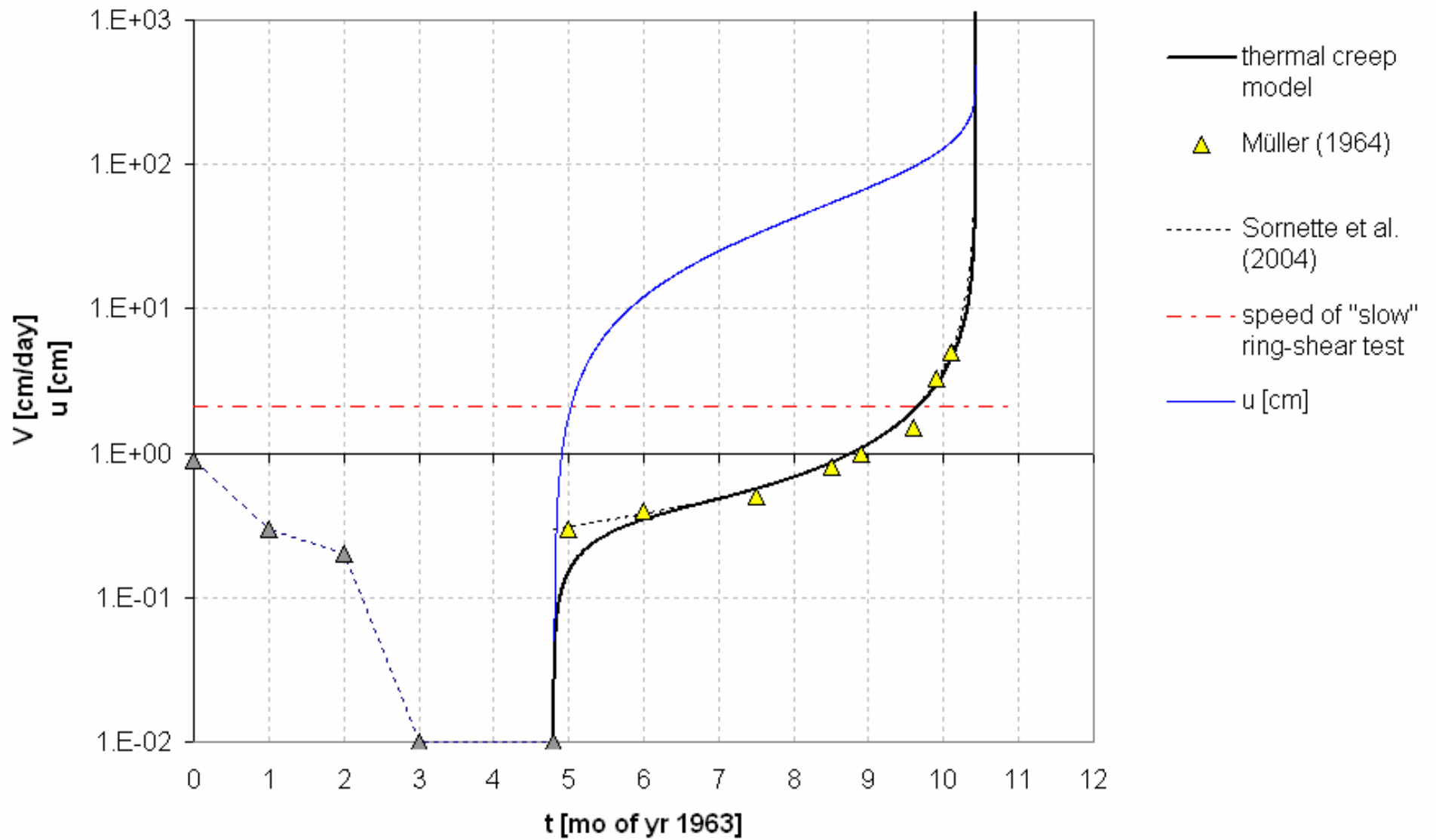
Since the catastrophic failure, a huge range of work has been undertaken on the causes of the failure. Initially there was a large amount of speculation about **the location of the sliding surface**, but more recent studies have confirmed that it **was located in thin (5 - 15 cm) clay layers** in the limestone. **Failure occurred in a brittle manner, inducing catastrophic loss of strength.** The speed of movement is probably the result of **frictional heating of the pore water in the clay layers** (Voight and Faust, 1982, 1992).

LECTURE NOTES - THE VAIONT LANDSLIDE , Dave Petley, 1999  
<http://www.sci.port.ac.uk/geology/staff/dpetley/imgs/enggeolprac/vaiont1.html>



**Müller-Saltzburg** (1964) in his Report “*The Rock Slide in the Vajont Valley*”, summarizes his impressions as follows:

*“...the interior kinematic nature of the mobile mass, after having reached a certain limit velocity at the start of the rock slide, must have been a kind of thixotropy. This would explain why the mass appears to have slid down with an unprecedented velocity which exceeded all expectations. Only a spontaneous decrease in the interior resistance to movement would allow one to explain the fact that practically the entire potential energy of the slide mass was transformed without internal absorption of energy into kinetic energy...Such a behaviour of the sliding mass was beyond any possible expectation; nobody predicted it and the author believes that such a behavior was in no way predictable...”.*



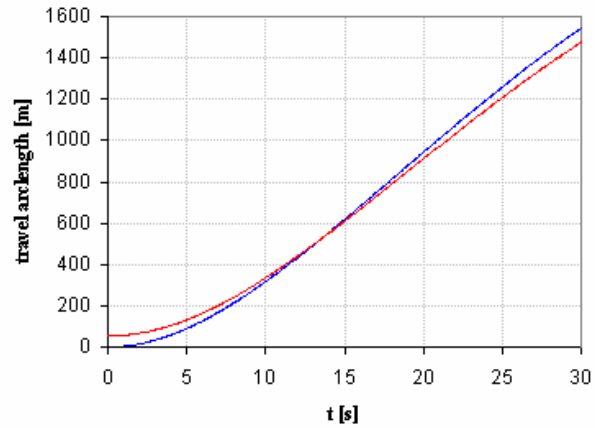
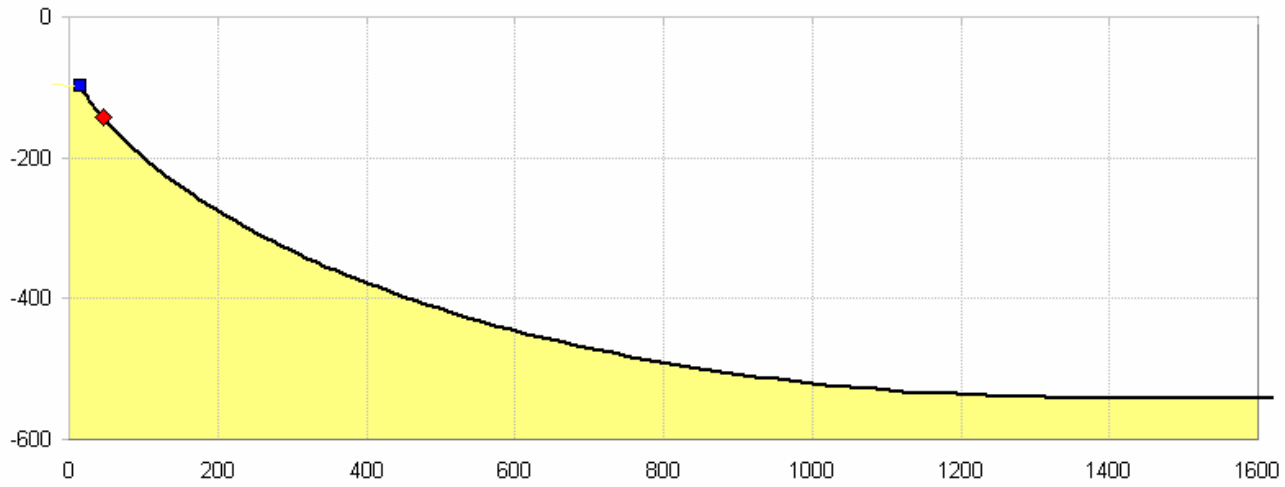
**Hendron, A.J. and Patton, F.D. (1985). The Vaiont slide, a geotechnical analysis based on new geologic observations of the failure surface. Technical Report GL-85-5. Washington, DC: Department of the Army US Corps of Engineers.**

**Veveakis, E., Vardoulakis, I. and Di Toro, G.(2007). Thermo-poro-mechanics of creeping landslides: the 1963 Vaiont slide, Northern Italy. *Journal of Geophysical Research*, in print.**

**Vardoulakis, I., 2002. Steady shear and thermal run-away in clayey gouges. *Int. J. Solids and Structures*, 39, 3831-3844**

**Vardoulakis, I. (2002). Dynamic thermo-poro-mechanical analysis of catastrophic landslides. *Géotechnique*, 52, (3),157-171.**

# Limitations of the simple sliding block model



Two neighboring blocks (blue and red in figure) released simultaneously, will soon collide, since the upper block will try to overtake the lower one. This means that the single block model is of relatively limited value. In reality the various rock masses which exist at various elevations will interact with lateral earth-pressure forces, which in turn will decelerate slightly the upper blocks and accelerate slightly the lower blocks, so that a common acceleration is established for the whole mass. This consideration leads inevitably to the so-called **flow-slides** models, which are derived from shallow-water type theories (Savage & Hutter 1989).

