International School LAndslide Risk Assessment and Mitigation
LARAM School 2008 (8-22 September, Ravello, Italy)
Session 1: Introduction to landslides: Landslide analysis using approaches based on: Geology, Geotechnics and Geomechanics

Basic Geodynamics of Landslides: The dynamic slip circle method

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The dynamic slip circle method (lagrangean)

The dynamics of landslide run-out

Flow-slides

I. The Dynamic Slip Circle Method

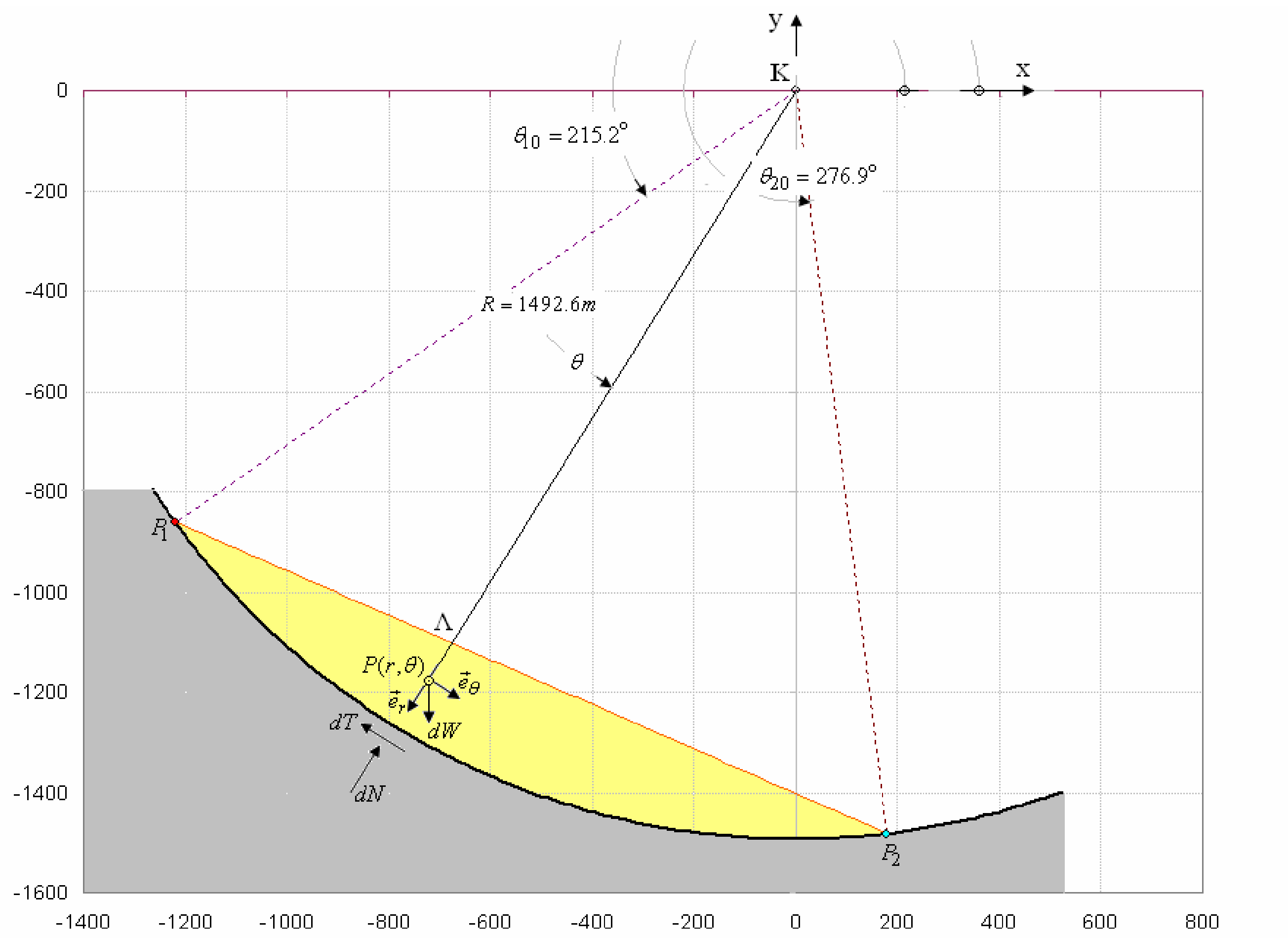
Taylor, D.W., *Fundamentals of Soil Mechanics*, Wiley, 1948.

Vardoulakis, I. (2002). Dynamic thermo-poro-mechanical analysis of catastrophic landslides. Géotechnique, 52, 157-171.

Initial phase of the slide motion: "rigid body" motion (Lagrangean approach)

$$\theta_i = \theta_{i0} + \int_0^t \omega(t)dt \quad (i = 1,2)$$

$$\omega = \frac{d\phi}{dt} \quad \Leftrightarrow \quad \phi = \int \omega dt$$



Kinematics:

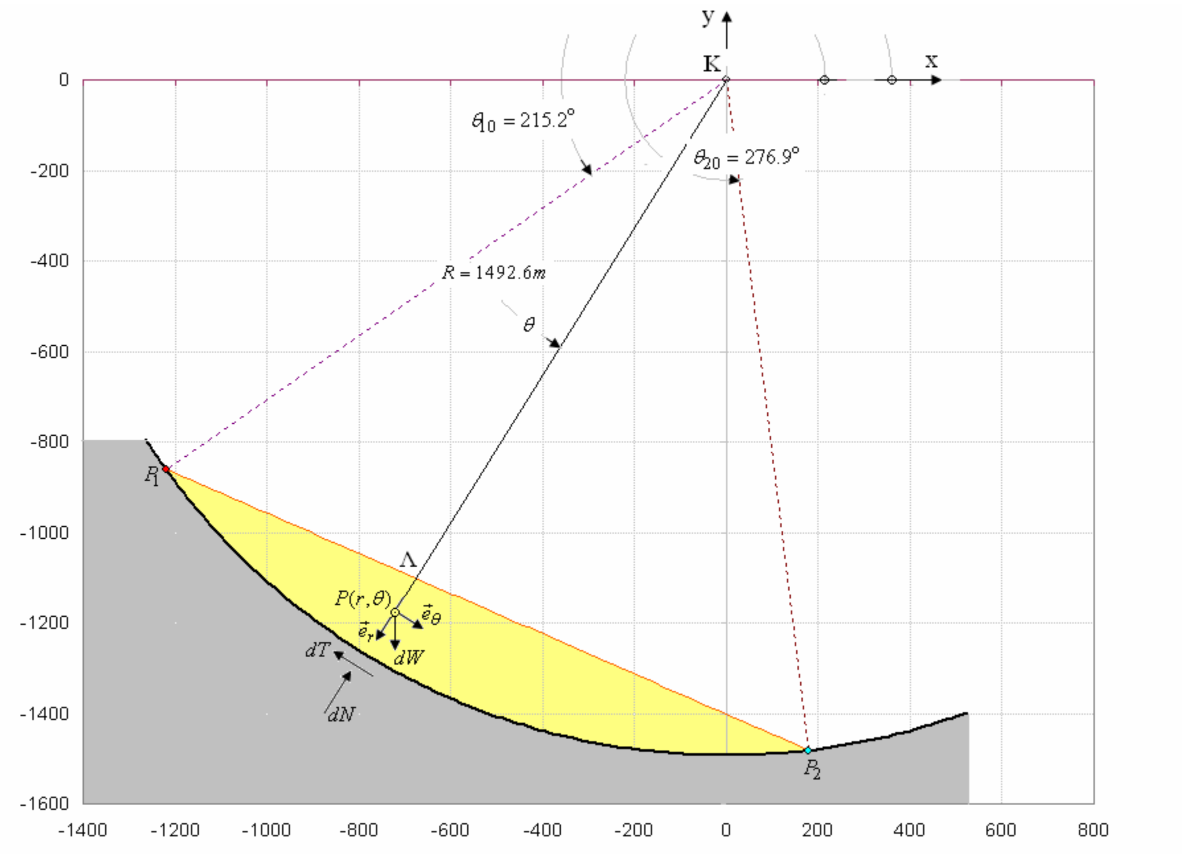
At any point we introduce the local polar ortho-normal basis vectors, in radial and tangential direction, respectively

$$\vec{R}_P = r\vec{e}_r$$

$$\vec{v}_P = v_\theta \vec{e}_\theta \quad , \quad v_\theta = r\omega$$

$$\vec{a}_P = a_r \vec{e}_r + a_\theta \vec{e}_\theta$$

$$a_r = -r\omega^2 \quad , \quad a_\theta = r\frac{d\omega}{dt}$$



Forces:

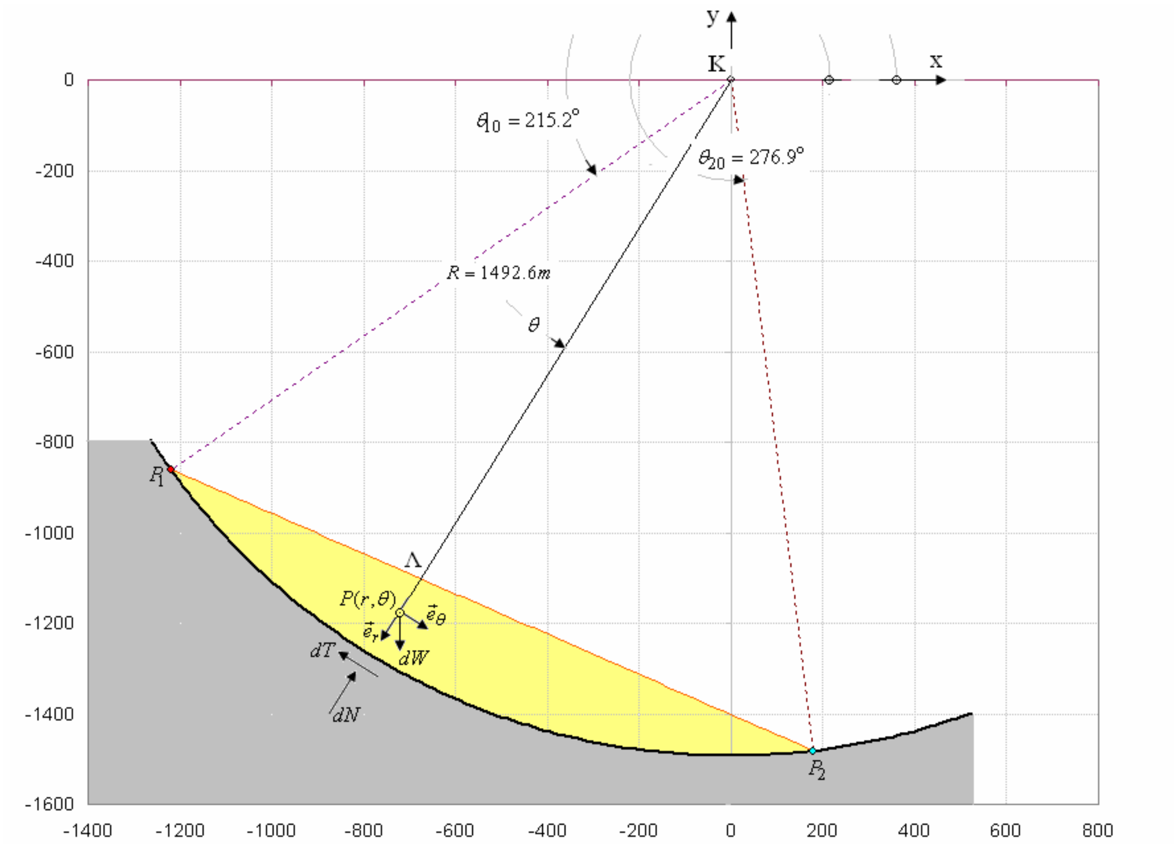
The self weight (body force) and the normal and shear reaction forces along the circular arc (surface forces)

$$d\vec{W} = dm\vec{g} = (\rho dV)\vec{g}$$

$$\vec{g} = -g\sin\theta\vec{e}_r - g\cos\theta\vec{e}_\theta$$

$$d\vec{N} = (-\sigma_n dS)\vec{e}_r$$

$$d\vec{T} = (-\tau_n dS)\vec{e}_\theta$$



The problem **is statically undetermined**, thus we need an **estimation of the dynamic normal reaction.** The normal reaction stress is determined via a dynamic consideration of the forces acting in radial direction:

$$\int_{(V)} d\vec{W}_r + \int_{(S)} d\vec{N}_r = \int_{(m)} \vec{a}_r dm \quad \Rightarrow$$

$$\int_{(V)} \rho g(-\sin\theta) dV + \int_{(S)} (-\sigma_n) dS = \int_{(m)} \rho(-r\omega^2) dV \quad \Rightarrow$$

$$\frac{R}{\rho g} \int_0^{2\alpha} \sigma_n d\alpha = \int_{\theta_1}^{\theta_2} \int_{R_1}^{R} (-\sin\theta) r dr d\theta + \frac{\omega^2}{g} I_p$$

$$I_p = \int_{\theta_1}^{\theta_2} \int_{R_1}^{R} r^2 dr d\theta$$ polar *surface* moment of inertia of the arc

The distribution of the normal reaction stress along the failure arc is undetermined. This is already known from the static analysis (cf. Taylor's, (1948) "friction circle method"). In order to reduce the failing earth body into an one-degree-of freedom "frictional pendulum", we assume that the normal reaction stress is **distributed uniformly** along the failing arc (Vardoulakis 2002) and we introduce the **mean value**,

$$\bar{\sigma}_n = \frac{1}{2\alpha} \int_0^{2\alpha} \sigma_n d\alpha \quad \Rightarrow$$

$$\frac{R}{\rho g} 2\alpha \bar{\sigma}_n = \int_{\theta_1}^{\theta_2} \int_{R_1}^{R} (-\sin\theta) r dr d\theta + \frac{\omega^2}{g} I_p$$

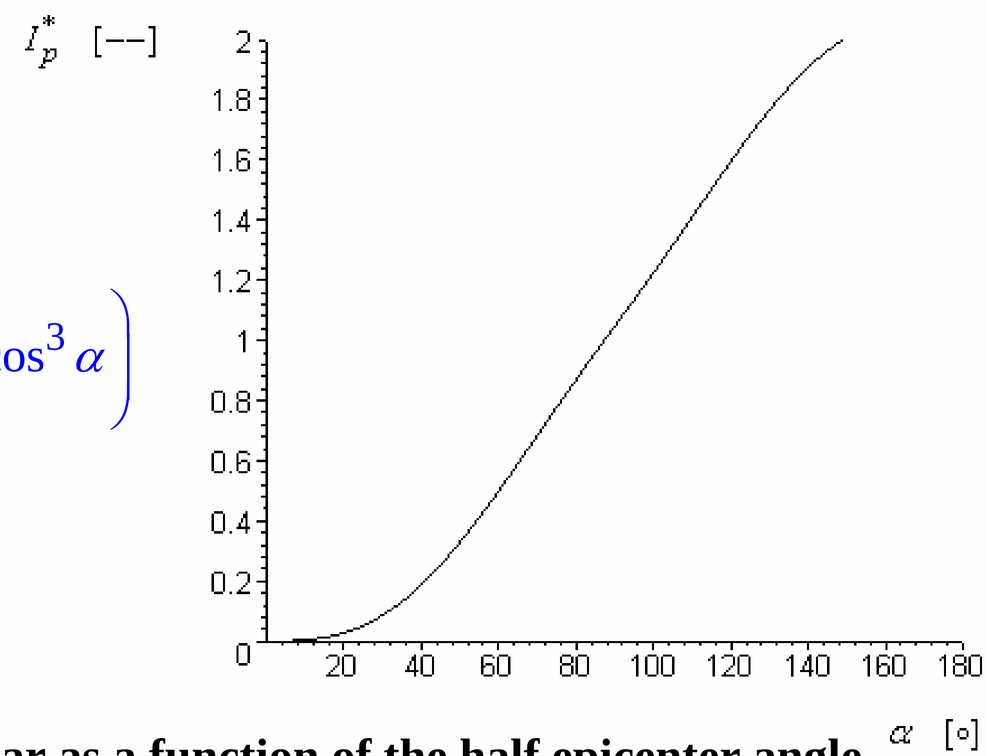
Non-dimensional variables

$$r^* = \frac{r}{R} \quad , \quad \bar{\sigma}_n^* = \frac{\bar{\sigma}_n}{\sigma_{ref}}$$

$$\sigma_{ref} = \rho g f \quad , \quad f = R\left(1-\cos\alpha\right)$$

$$\Rightarrow \quad I_p = R^3 I_p^*$$

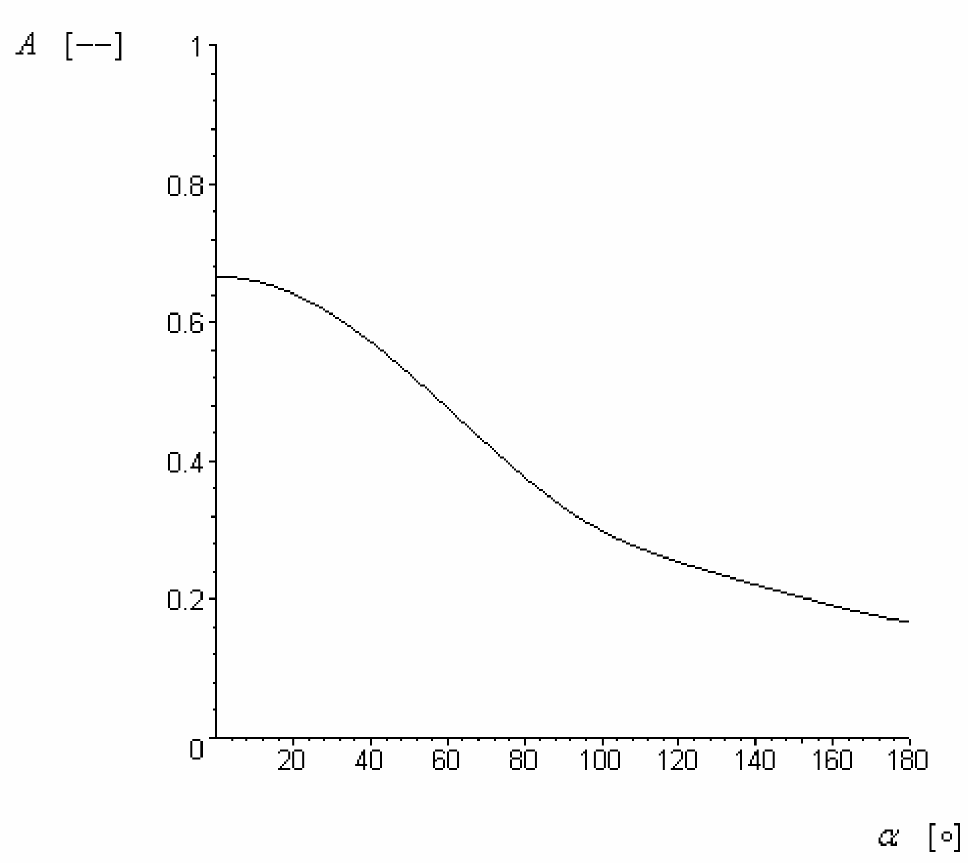
$$I_p^* = \frac{1}{3}\left(2\alpha - \frac{1}{2}\sin 2\alpha - \ln\left(\left|\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right|\right)\cos^3\alpha\right)$$



Polar moment of inertia of circular as a function of the half epicenter angle

$$\frac{R}{\rho g} 2\alpha \bar{\sigma}_n = \int_{\theta_1}^{\theta_2} \int_{R_1}^{R} (-\sin\theta) r dr d\theta + \frac{\omega^2}{g} I_p \quad \Rightarrow \quad \bar{\sigma}_n^* = \bar{\sigma}_{n,st}^* + A\omega^{*2}$$

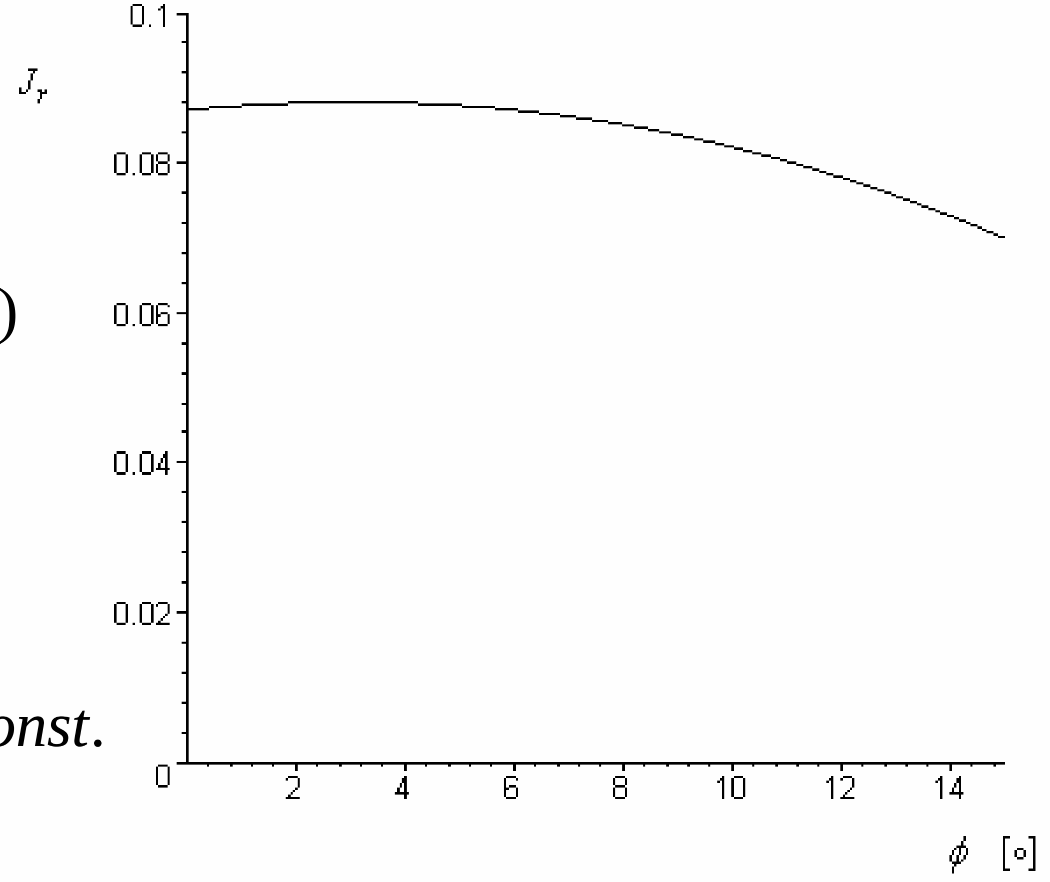
$$A = \frac{I_p^*}{2\alpha(1-\cos\alpha)}$$



$$\bar{\sigma}_n^* = \bar{\sigma}_{n,st}^* + A\omega^{*2}$$

$$\bar{\sigma}_{n,st}^* = \frac{1}{2\alpha\left(1-\cos\alpha\right)} J_r(\theta_1,\theta_2)$$

$$J_r = \int_{\theta_1}^{\theta_2}\int_{r_1^*}^{1} r^*(-\sin\theta)dr^*d\theta \approx const.$$



Normal reaction estimate

$$\bar{\sigma}_n^* = \bar{\sigma}_{n,0}^* + A\omega^{*2}$$

The static value (static equilibrium) is augmented by a dynamic term that is due to the centripetal acceleration

$$a_r = -r\omega^2$$

Dynamics (balance of angular momentum)

$$\Theta \frac{d\omega}{dt} = M^K$$

$$\Theta = \int_{(m)} r^2 dm = \int_{(V)} r^2 \rho dV = \ell \rho \int_{\theta_1}^{\theta_2} \int_{R_1}^{R} r^3 dr d\theta$$ **(mass moment of inertia)**

$$M^K = M_W^K + M_T^K = \int_{(V)} r dW_\theta - \int_{(S)} R dT$$

(moment of forces with respect to the pole K)

$$M^K = M_W^K + M_T^K$$

$$M_W^K = \int_{(V)} r\rho g\left(-\cos\theta\right) dV = \ell\rho g \int_{\theta_1}^{\theta_2} \int_{R_1}^{R} r^2 \left(-\cos\theta\right) dr d\theta$$

$$M_T^K = \int_{(S)} \mu\sigma_n dS = \mu(t)\bar{\sigma}_n \ell R 2\alpha$$

$$\bar{\sigma}_n = \sigma_{ref}\left(\bar{\sigma}_{n,st}^{*} + A\omega^{*2}\right)$$

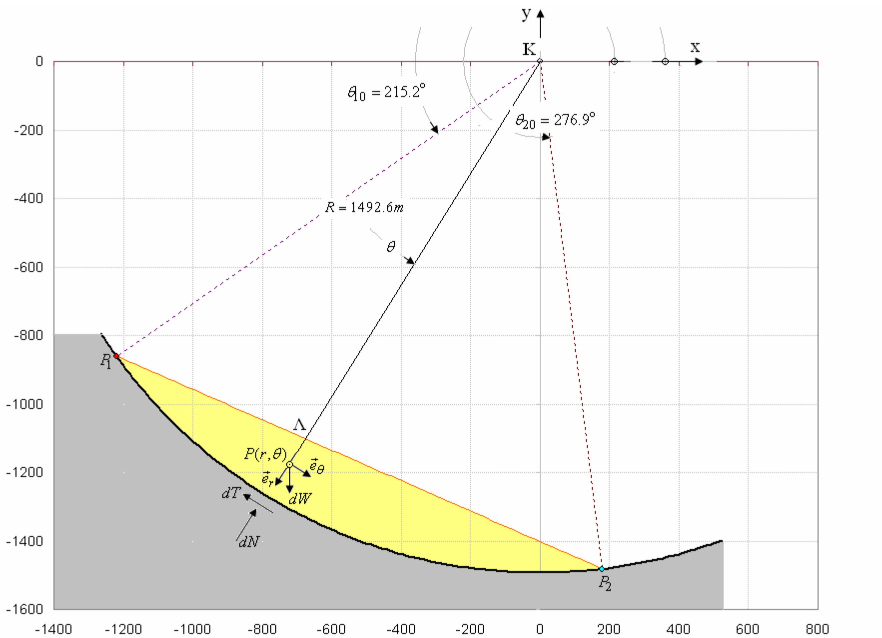
Non-dimesinalization

$$t^* = \frac{t}{t_c} \quad , \quad t_c = \sqrt{\frac{R}{g}} \quad \Rightarrow \quad \Theta^* \frac{d\omega^*}{dt^*} = J_\theta - \lambda \bar{\sigma}_n \mu(t)$$

$$\Theta^* = \int_{\theta_1}^{\theta_2} \int_{R_1^*}^{1} r^{*3} dr^* d\theta \quad , \quad J_\theta = \int_{\theta_1}^{\theta_2} \int_{R_1^*}^{1} r^{*2} \left(-\cos\theta\right) dr^* d\theta$$

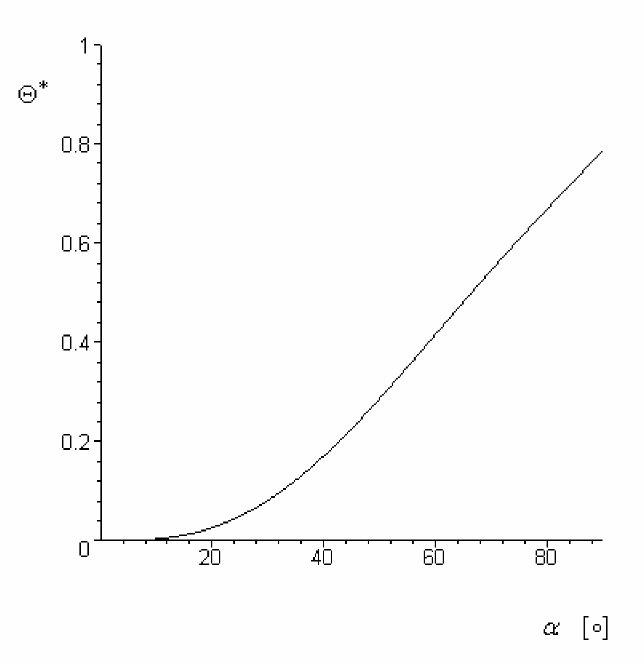
$$R_1^* = (K\Lambda)/R$$

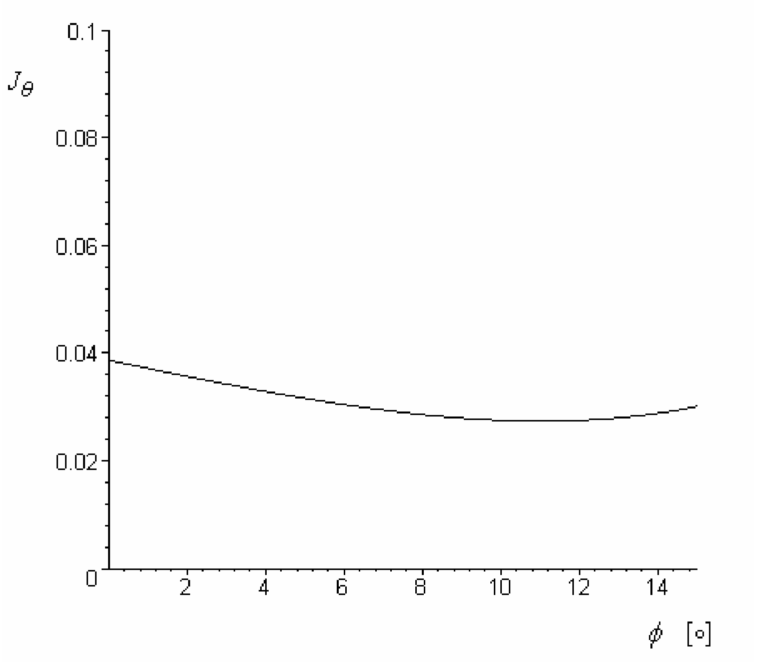
$$\lambda = 2\alpha\left(1 - \cos\alpha\right)$$



$$\Theta^* \frac{d\omega^*}{dt^*} = J_\theta - \lambda\left(\bar{\sigma}^*_{n,0} + A\omega^{*2}\right)\mu(t^*)$$

$$\Theta^* = \frac{1}{2}\left(\alpha - \frac{1}{6}\sin 2\alpha\left(1 + 2\cos^2\alpha\right)\right)$$





The governing equation

$$\frac{d\omega^*}{dt^*} = a - b\omega^{*2}$$

$$a = \frac{1}{\Theta^*}\left(J_\theta - J_{r0}\mu(t)\right)$$

$$b = \frac{\lambda A}{\Theta^*}\mu(t)$$

$$\frac{d\omega^*}{dt^*} = a - b\omega^{*2} \quad and \quad \omega^*(0) = 0 \quad \Rightarrow$$

$$\omega^* = \sqrt{\frac{a}{b}} \tanh\left(\sqrt{ab} t^*\right)$$

The motion starts as soon as a trigger mechanism has reduced the friction coefficient to a value that is less then the corresponding **limit equilibrium** value,

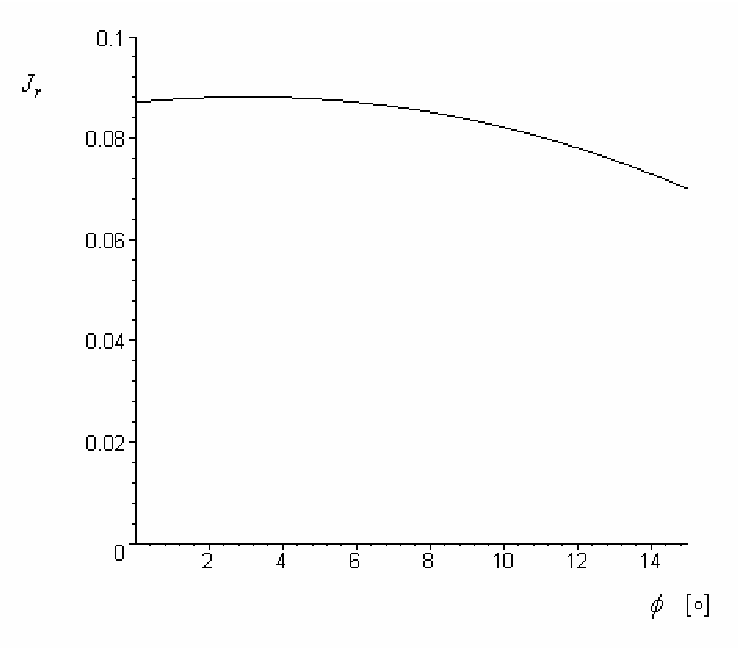
$$\omega^* = 0 \quad \Rightarrow \quad a = \frac{1}{\Theta^*}\left(J_\theta - J_{r0}\mu\right) = 0 \quad \Rightarrow$$

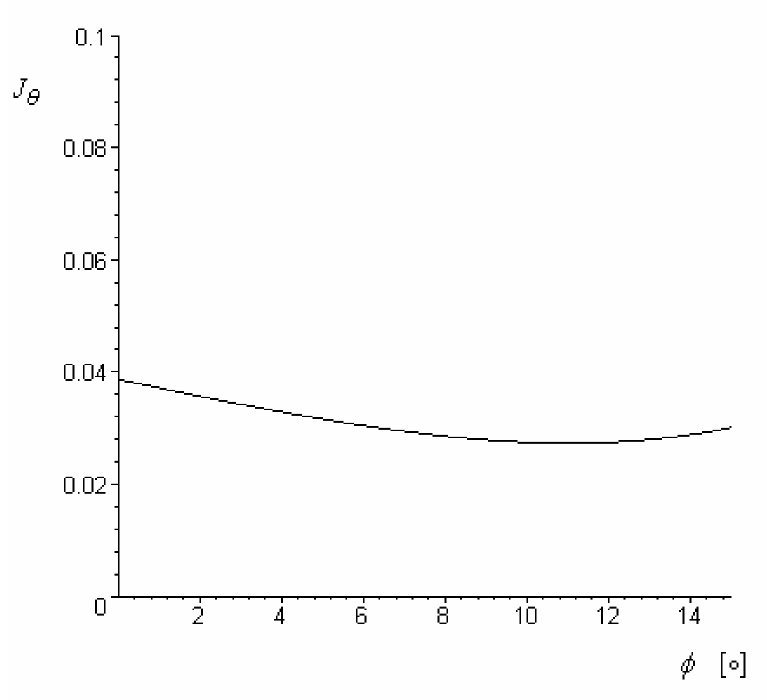
$$\mu_{eq} = \frac{J_{\theta 0}}{J_{r0}}$$

Computational Example

Table 2-1: Geometric and mechanical properties of the assumed circular arc (cf. Figure 1-1)

R	γ	θ_{10}	θ_{20}	2α	I_p	σ_{ref}	$\bar{\sigma}_{n,0}$	A
$[m]$	$[kN/m^3]$	$[\circ]$	$[\circ]$	$[\circ]$	$[km^3]$	$[MPa]$	$[MPa]$	$[--]$
1492.6	20	215.2	276.9	61.7	0.308	4.223	2.413	0.468

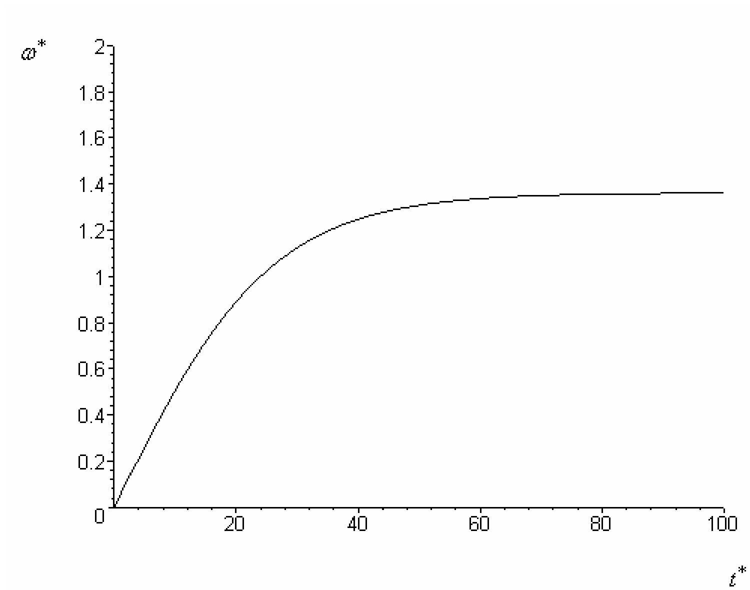




$$\mu_{eq} = \frac{J_{\theta 0}}{J_{r0}} = \frac{0.039}{0.087} = 0.444$$

Table 4-1: Dimensionless parameters entering the governing Eq. (3.11); cf. Table 2-1.

Θ^*	$\sim J_r$	$\sim J_\theta$	λ	$\bar{\sigma}^*_{n,0}$	A	$\varphi_{C,eq}$
0.4376	0.0870	0.0387	0.1523	0.5713	0.468	24°



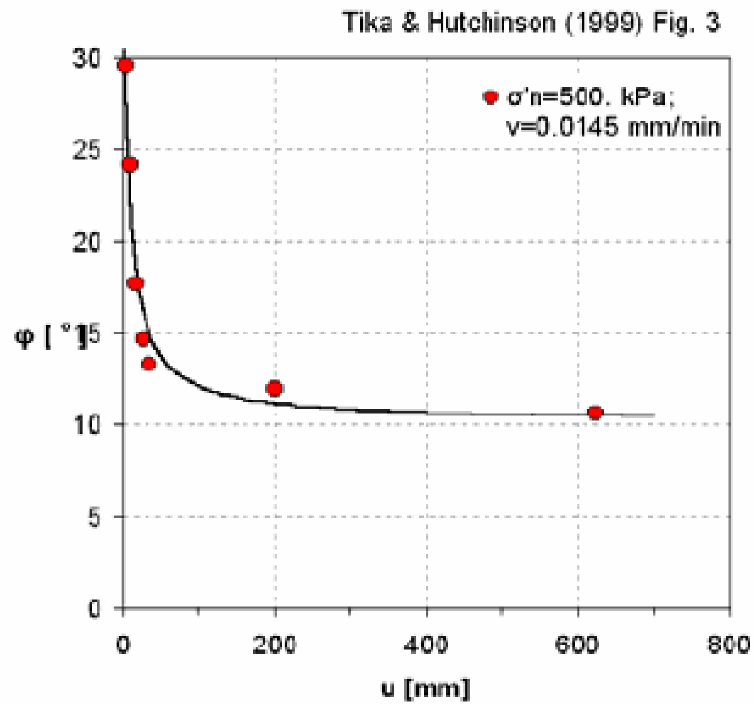
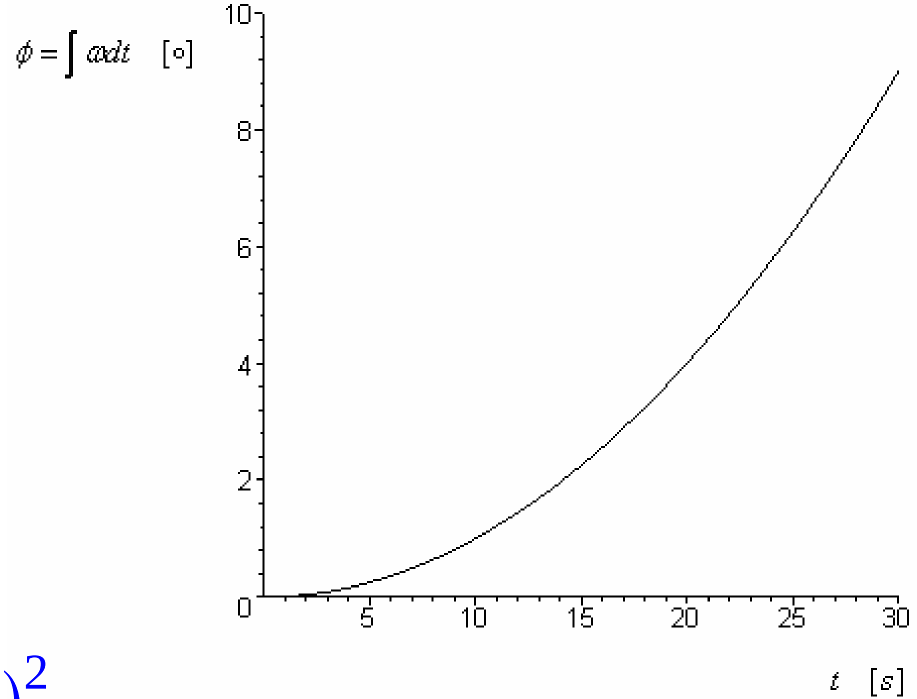


Figure 4-1: Friction-displacement softening in ring-shear tests after Tika and Hutchinson (1999).

$$\omega = c_0 \tanh\left(c_2 t\right) \quad , \quad c_0 = 0.11\text{sec}^{-1} \quad , \quad c_2 = 0.003\text{sec}^{-1}$$

$$\phi = \frac{c_0}{c_2} \ln\left(\cosh\left(c_2 t\right)\right)$$

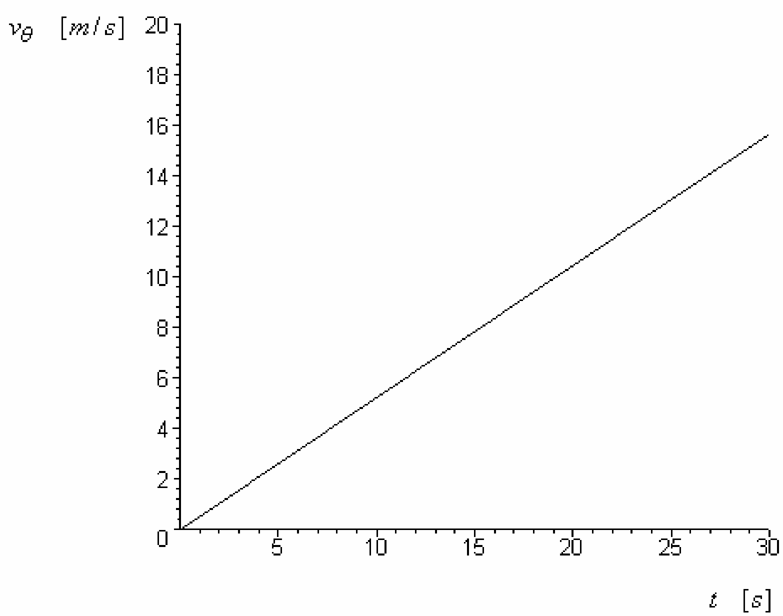


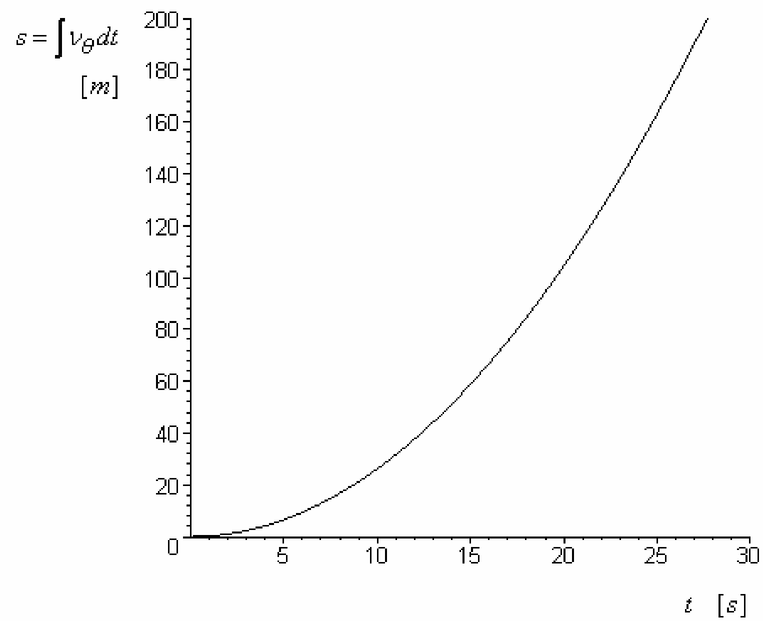
$$v_\theta = R\omega(t) = c_1 \tanh\left(c_2 t\right) \approx c_1 c_2 t$$

$$c_1 \approx 165\text{m/sec}$$

$$c_1 c_2 \approx 0.5 m \sec^{-2}$$

$$s = \int v_\theta dt = c_1 \ln\cosh\left(c_2 t\right) \approx c_1 \frac{1}{2}\left(c_2 t\right)^2$$





Notably that the assumed severe reduction of the friction angle from its equilibrium value to its residual value is leading in the considered short time interval to large velocities and displacements!